# A Chaos and Fractal Dynamic Approach to the Fracture Mechanics

Lucas Máximo Alves[1] and Rui F.M. Lobo[2]

[1] GTEME - Grupo de Termodinâmica, Mecânica e Eletrônica dos Materiais, `lmalves@uepg.br`,
Departamento de Engenharia de Materiais, Universidade Estadual de Ponta Grossa - Caixa Postal 1007, Av. Gal. Carlos Calvalcanti, 4748, Campus UEPG/Bloco L - Uvaranas, Ponta Grossa, Paraná,CEP. 84030.000 Brasil.

[2] Grupo de Nanotecnologia e Ciência a Nano-Escala, `rfl@fct.unl.pt`,
Departamento de Física, Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa, 2829-516 Caparica – Portugal.

**Summary.** It is shown that the onset of instabilities observed in the fracture of brittle isotropic materials is a consequence of the mathematical structure of chaos that underlies such phenomena. The straight line crack velocity is written in the form of a logistic map explaining the onset of instabilities observed by Fineberg et al. This approach provides a single and concise tool to study this and other nonlinear aspects presented by dynamic crack growth.

## 1 Introduction

Dynamic fracture has been stimulating a growing interest not only because of its fundamental importance in understanding fracture processes but also because of the challenges to mathematical analysis and experimental techniques.

Fineberg and co-workers performed experiments on fast crack growth [1,2] for different brittle materials, such as PMMA, soda-lime glass, etc., revealing many new aspects, defiant to the fracture dynamic theory, related to unstable crack growth. They observed the existence of a critical velocity starting from which instability begins and measured the correlation between the fluctuations in crack growth velocity and the ruggedness of the generated surfaces. They found a time delay (of the order of the stress relaxation time) present during the whole fracture process in the materials used by them. In spite of this experimental observation that can be associated to stress relaxation properties in viscoelastic materials, they were unable to relate it physically to the onset of the instability phenomenon itself. Therefore the need for a correct mathematical description of the instability process in crack growth represent one of the interesting challenges in dynamic fracture.

On the other hand, Kostrov–Nikitin [3] and Christensen [4] modelled the quasi-static crack growth case in viscoelastic materials showing that the stress field at the crack tip does not depend on the relaxation properties of the material but that it is determined by the instantaneous elastic modulus at the crack tip. Similarly, analogous considerations have been taken and extended to dynamic case [6, 7, 7]. Analyzing this last situation Freund [8] admits that there is really a paradox because the process zone in front of the crack tip depends strongly on the loading conditions and usually ideal boundary conditions are considered, such as a thin crack tip describing a punctual fracture in a short range effect in the obtaining of asymptotic solutions to the problem. Therefore, such paradox is solved by introducing a separation zone of finite extend in the form of a cohesive flaw zone of some length, say $l_o$, ahead of the moving crack tip [8, 9] to separate the crack formed from the instantaneous process at the crack tip. Such cohesive zone accompanies the crack growth process at the crack tip, while the crack opens gradually against the resistance of some cohesive stress within this zone. The time, $t$, required for a crack tip to advance a distance equal to $l_o$, as the crack grows at crack growth velocity, $v_o$, introduces a process time, given by $t = l_o/v_o$, that must be compared to a characteristic relaxation time, $\tau \sim t$, of the material to determine whether the process is "fast" or "slow" [8].

Actually, if it is considered that the relaxation time, $\tau$, is much smaller than the time, $t$, taken by the process zone to move forward the crack, the zone is stable and the relaxation effects don't affect the process as it is shown for the quasi-static case ($v \to 0$) [3] by the results of Kostrov–Nikitin and Christensen [4]. But for fast crack growth the relaxation time $\tau$ is very large compared to the time, $t$, taken by the process zone to move a distance $l_o$. This happens when the crack growth velocity tends to the Rayleigh waves velocity of the material. In this case the relaxation effects become important and the crack becomes unstable as shown experimentally by Fineberg and co-workers [1, 2] by means of the correlation measured between the fluctuation in crack growth velocity and the ruggedness of the generated surfaces. In these correlation measurements they found a time delay between the oscillations in the crack growth velocity and the surface profile of approximately 3 µs and 1.0 µs for PMMA and soda-lime glass respectively. The critical velocities at which the instability begins are 340 m/s and 1100 m/s, respectively, and both values correspond to 0,34 of the respective Rayleigh surface wave velocity, $c_R$, in these materials, whose values are 975 m/s and 3370 m/s for PMMA and soda-lime glass respectively. This means that at these velocities the characteristics lengths of the process zone is of the order of $l_o = 3\,\mu\text{s} \times 340\,\text{m/s} \sim 1,02\,\text{mm}$ and $l_o = 1.0\,\mu\text{s} \times 1100\,\text{m/s} \sim 1.10\,\text{mm}$ for the two materials cited above, also agreeing with other experimental results found by Fineberg and co-workers.

In view of these results there are two alternatives to be considered: either the fast crack growth process is described by using the stress field modelled with a retardation time that takes into account the effects of relaxation of the material, or the mathematical description of a process zone is used that keeps all the information of delay produced by the relaxation of the material, leaving unsolved the instability process, and the previous case is ignored. Historically the latest option has been the solution proposed by Irwin and other scientists to describe the complex effect of the plasticity and viscoelasticity at the crack tip [10–12]. It is not a surprise that, when intending to study a Mechanics of the Fracture that can be universally applied to any kind of material and to any mode of loading, it should receive another formulation.

In this chapter, in order to explain Fineberg et al. results [1,2] one makes use of a physical model in which the retardation time is introduced in an explicit way generalizing the classical formulation of Fracture Mechanics. It will also be shown that this mathematical procedure will include the fracture process inside the family of phenomena described by nonlinear dynamic processes with the advantage of using the whole mathematical development so far accomplished for these processes and the classic formulation of Fracture Mechanics generalized by the explicit inclusion of time retardation.

## 2 Theoretical Development of a Chaotic Model to Dynamic Fracture

### 2.1 The Fast Crack Growth in the Fineberg–Gross Experiments

Consider a semi-infinite plane plate under Mode-I loading and plane strain in elastodynamic crack growth conditions, as shown in Fig. 1. The experimental configuration of the body under testing is equivalent to a infinite plate condition to avoid the edges effects on the crack growth process. In this experiment a fast crack growth develops as a result of a high loading rate and a high strain rate in the dynamic fracture process. In agreement with

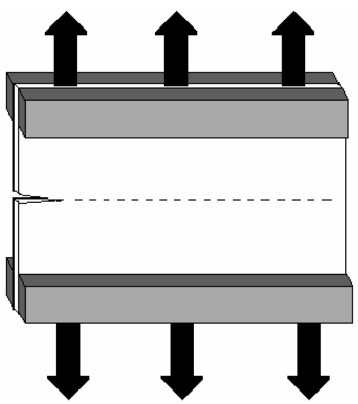

**Fig. 1.** Experimental set up of a plane plate under Mode I loading in dynamic fracture process in accord to Fineberg–Gross [1,2] experiments

Fineberg–Gross [1, 2] the aspect of the fracture surfaces is rugged and the crack growth velocity shows a instability from a critical velocity.

In their experiments [1, 2] Fineberg–Gross showed that when the crack speed reaches a critical value a strong temporal correlation between the velocity, $v_o(t)$, and the response in the form of the fracture surface at $A_o(t+\tau)$ takes place (having its notation changed, in the present text, to $L_o(t)$ instead of $A_o(t)$ to designate the fracture surface length). The time delay measured between this two magnitudes presents a value $\tau$ of about $\simeq 3\,\mu$s for PMMA and 1.0 μs for soda-lime glass, for example, showing that there is a given value for each material.

## 2.2 The Origin of the Time Delay from Fineberg–Gross Experimental Evidences

Crack growth is a complex process that involves many physical phenomena (bond breaking, sound and light emission, heat generation, etc.) which participate in the crack growth process. Also, the relative importance of each one depends on the conditions under which crack growth takes place. It is well known that for any given material a certain time delay exists between the loading and the crack growth, at the beginning of the crack growth, in experiments performed under time-dependent loading conditions [8]. Moreover, this time delay tends to disappear with the crack growth [8]. However, since dynamic fracture involves heat generation it is necessary to take into account the fact that a viscoelastic material such as the one used by Fineberg–Gross [1, 2] shows viscoelastic relaxation phenomena. This is so because the heat developed at the crack tip might have changed the local properties of the material making the hypothesis of viscoelastic relaxation very plausible. In the case of a viscoelastic material, such as PMMA, if a considerable amount of viscoelastic material is formed at the crack tip (size of the order of $l_o \sim v_o\tau$), the time delay appears locally remaining because of the existence of the persistent creep phenomenon that takes place at fast dynamic crack growth, as it is evident by the correlation shown in the Fineberg–Gross experiments [1,2]. Therefore, this may explain the phenomenology behind the time delay obtained by Fineberg et al. experiments [1, 2]. Taking into account this strong evidence of time delay between the two magnitudes mentioned above the fast dynamic crack growth can be expressed by the elastodynamic energy release rate, $G_{oD}$, at a given time such as it does not depend on the crack velocity at the same moment [12–15] as shown in Fig. 2 and as it will be shown below.

The time delay obtained by Fineberg and co-workers [1, 2] attributed to the viscoelastic properties of the material at the crack tip, possibly develops in several steps. In the case of the vitreous (glasses) and polymeric materials the viscoelastic properties of the material at the crack tip become evident because at fast crack growth heat develops at crack tip, and due to the poor heat conduction, temperature rises with possible softening of this material, (in front of the crack tip) as explained above, even at room temperature. In

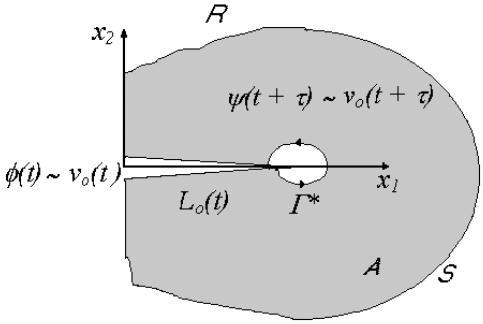


**Fig. 2.** Schematic crack tip region with time delay between the input flux energy, $\phi(t)$ and output flux energy, $\psi(t+\tau)$

the case of metals, due to higher heat conduction, the viscoelastic properties only appear when the sample is tested at temperatures greater than thirty percent of their absolute melting temperatures [11].

"There are two competing mechanisms involved in the crack growth that characterize the creep deformation. The blunting of the material in front of the crack tip relaxes the crack tip stress field and tends to retard crack growth. The other mechanism results in an accumulation of creep damage in the form of microcracks and voids that enhance crack growth as they coalesce" [11].

### 2.3 The Foundations of Quasi-Static and Dynamic Fracture Mechanics

For quasi-static crack growth the usual elastic energy release rate, $G_o$, is given by [12]

$$G_o \equiv \frac{\mathrm{d}(F - U_{Lo})}{\mathrm{d}L_o} , \tag{1}$$

where $F$ is the work performed by external forces on the sample, $U_{Lo}$ is the change in elastic strain energy caused by the introduction of a crack with length, $L_o$, into the sample. $L_o$ is the distance between two points of the crack (or projected crack length) and the subscript zero denotes the plane projected magnitudes.

The Griffith–Irwin energy balance approach for stable fracture requires that [10, 16]

$$G_o \geq R_o , \tag{2}$$

where $R_o$ is the crack resistance per unit thickness defined as

$$R_o \equiv \frac{\mathrm{d}U_{\gamma o}}{\mathrm{d}L_o} , \tag{3}$$

and $U_{\gamma o}$ is the product of the specific elastic surface energy of the material, $\gamma_{o(\mathrm{eff})}$, by the projected surface area of the crack (two surfaces, length $L_o$), i.e., the plane surface area of the crack with actual (rugged) crack length $L$ and unit width. It is important to stress that $L$ refers to the actual (rugged) crack length and $L_o$ to the projected crack length of $L$, i.e., a distance between two points [17]. Therefore

$$U_{\gamma o} = 2L_o \gamma_{o(\text{eff})} , \tag{4}$$

and according to Alves [17] the constant crack growth resistance, $R_o = 2\gamma_{(\text{eff})}$ needs to be corrected by the ruggedness in the follow way

$$R_o = 2\gamma_{\text{eff}} \frac{\mathrm{d}L}{\mathrm{d}L_o} , \tag{5}$$

where the mathematical term $\mathrm{d}L/\mathrm{d}L_o$ was called the local ruggedness of the fracture surface [17]. From the quasi-static Griffith criteria, the condition to trigger the beginning of crack growth, is given by $G_o = R_o$. Gao [18] introduced the role of the surface roughening and branching instabilities in dynamic fracture proposing a so-called "wavy-crack model" motivated by experimental observations where rapidly moving cracks develop roughened fracture surfaces. The essence of his model consists in separating the microscopic crack-tip motion with local velocity, from the macroscopically observable crack motion with apparent velocity.

Similarly for the stationary case the elastodynamic energy release rate, $G_{oD}$, is given by [12]

$$G_{oD} \equiv \frac{\mathrm{d}[F - (U_{Lo} + T_o)]}{\mathrm{d}L_o} , \tag{6}$$

and $T_o$ is the kinetic energy of the crack growth which can be written as:

$$T_o = \frac{1}{2}\rho v_o^2 \int\int \left[ \left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2 + \left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 \right] \mathrm{d}x \; \mathrm{d}y . \tag{7}$$

The velocities indexed by $o$, as $v_o$, refers to the growth rate of the projected crack length, $L_o$. This equation will be not considered in the calculations that will be followed here; it is just shown up in the text in order to form the body of the classic considerations, that cannot explain the phenomenon in question. A more general expression was included, however, the calculations do not require the explicit use of the expression of the kinetic energy, $T_o$, but general principles will just be used.

Mott in 1948 [19] proposed that all excess of the elastodynamic energy release rate, $G_{oD}$, above the energy necessary to create the fracture surfaces, $2\gamma_{\text{eff}}$, is transformed into kinetic energy of crack growth. However, it is possible to generalize this proposition to the case of a material which shows a non-constant crack growth resistance, $R = 2\gamma_{\text{eff}}$, that develops a ruggedness such as that of the kind $\mathrm{d}L/\mathrm{d}L_o$ expressed in Alves [17]. Rewriting (6):

$$G_{oD} = \frac{\mathrm{d}(F - U_{Lo})}{\mathrm{d}L_o} - \frac{\mathrm{d}T_o}{\mathrm{d}L_o} \ . \tag{8}$$

The first term on the right side of (8) corresponds to the usual elastic energy release rate, $G_o$ given by (1), and so:

$$G_{oD} = G_o - \frac{\mathrm{d}T_o}{\mathrm{d}L_o} \ . \tag{9}$$

This equation shows the relationship between $G_o$ and the elastodynamic energy release rate, $G_{oD}$. In analogous way to (2), the Irwin–Mott condition of dynamic crack growth is given by

$$G_{oD}(L_o, v_o) \geq \Gamma_0(L_o, v_o) \ , \tag{10}$$

where $\Gamma_0 = \Gamma_0(L_o, v_o)$ is the dynamic crack growth resistance, now depending on the crack growth length, $L_o$, and velocity, $v_o$.

In agreement with Griffith fracture criterion for the quasi-static case it is possible to write, in analogous way, the elastodynamic crack growth condition, as follows

$$G_{oD}(L_o, v_o) = \Gamma_0(L_o, v_o) \ , \tag{11}$$

where $\Gamma_0$ can be write as $\Gamma_0(L_o, v_o) = R_o - \mathrm{d}T_o(L_o, v_o)/\mathrm{d}L_o$ using (5) for crack resistance, $R_o$.

The expression (6) mentioned here will not be used in the same way, but completes the idea to be explained in the following section by using a variable separation in the non steady-state case, which is a quite knew procedure.

### 2.4 Advanced Dynamic Fracture Mechanics Considerations

The literature [13] points to the fact that experimental results confirm the continuum theory of dynamic brittle fracture for fast cracks, where expressions similar to 6 are used to explain the phenomena in focus. On the other hand, in classical dynamic fracture mechanics it is usually assumed that rupture is controlled by the expression [8]

$$G_{oD} \simeq G_o \left(1 - \frac{v_o}{c_R}\right) = 2\gamma_{\mathrm{eff}} \ . \tag{12}$$

This expression equating $G_{oD}$ with $v_o$ is only true locally, i.e., in the immediate vicinity of the crack tip. For the configuration of plane stress applied to a medium of infinite extent, for example, a crack will constantly accelerate.

For this case, the quasi-static energy release rate, $G_o = \pi\sigma_0^2 L_o/2E_o$, in (12), is proportional to the fault crack length, $L_o$, so that as the rupture grows, $L_o$ grows, and the velocity, $v_o$, tends to $c_R$ in order to maintain $2\gamma_{\mathrm{eff}}$ at a constant value. This formula fails because, accordingly to the well established fracture mechanics theory, as the velocity increases, $2\gamma_{\mathrm{eff}}$ ceases to be a constant and depends on rupture speed. At velocities near 0.4 or 0.5 of $c_R$, rupture branches appear and (12) ceases to be applicable [3,20,21]. This explanation was well known to Kostrov, Slepyan, Knauss and Ravi–Chandar, Rice, etc and was rediscovered by physicists in the early 1990s [1,2,13,22,23]. In the dynamical case Slepyan [8,20] proposed that the elastodynamic energy release rate, $G_{oD}(L_o, v_o)$, can be written in terms of the elastic energy release rate, $G_o(L_o)$, for the stable case [16] in the following way:

$$G_{oD}(L_o, v_o) = G_o(L_o) g\left(\frac{v_o}{c_R}\right) . \tag{13}$$

where $c_R$ is the Rayleigh velocity.

To write the functional dependence of this elastodynamic energy release rate, $G_{oD}(L_o, v_o)$, Slepyan [20] propose a Maximum Energy Dissipation Principle. In Alves [24] one has also demonstrated the need for correcting this principle in order to include the ruggedness of fracture surface [17]. For the Fineberg–Gross [1,2] experimental set up of a semi-infinite plane plate under Mode I loading, the elastodynamic crack growth condition, given by (11) yields:

$$G_{oD}(L_o, v_o) = \frac{2\gamma_{\text{eff}}(\mathrm{d}L/\mathrm{d}L_o)}{1 - \frac{v_o}{c_R}(\mathrm{d}L/\mathrm{d}L_o)} = \Gamma_0(L_o, v_o) , \tag{14}$$

Equation (14) shows a great agreement with Gross experimental results [22] as was fitted by Alves [24].

Using the Maximum Energy Dissipation Principle of Slepyan [20], modified to include the ruggedness, $\mathrm{d}L/\mathrm{d}L_o$ of fracture surface, and the Irwin–Mott crack growth dynamic condition given by (10) in (14), including (13), one can assert that:

$$G_o(L_o) g\left(\frac{v_o}{c_R}\right) \geq \frac{2\gamma_{\text{eff}}(\mathrm{d}L/\mathrm{d}L_o)}{1 - \frac{v_o}{c_R}(\mathrm{d}L/\mathrm{d}L_o)} . \tag{15}$$

Comparing (14) with (13) it can be concluded in agreement with Alves [17] that for the brittle materials case where $J \equiv G$, that:

$$G_o(L_o) \geq 2\gamma_{\text{eff}} \frac{\mathrm{d}L}{\mathrm{d}L_o} , \tag{16}$$

and

$$g\left(\frac{v_o}{c_R}\right) \sim \frac{1}{1 - \frac{v_o}{c_R}} . \tag{17}$$

This is an equation that shows dependence on the particular experimental set up under use. The case studied here is of a plane plate under Mode I loading and boundary conditions of infinite body accordingly to Freund [25].

## 2.5 The Foundations of Non-Stationary Dynamic Fracture Mechanics

For, for the non-stationary case the dynamic energetic balance for the bi-dimensional case can be write written as

$$P = \dot{U} + \dot{T} + \phi \ , \tag{18}$$

where $\phi$ is the rate of work done by the traction on the surface $S$, $\dot{U}$ is the rate of increase of the strain energy, $\dot{T}$ is the kinetic energy in the region with $R$ with area $A$ and $F$ is the energy flux into the crack-tip region.

$$P = \int_S T_i \dot{u}_i \ \mathrm{d}s \ , \tag{19}$$

$$U = \lim_{\Gamma^* \to 0} \int_R W(\epsilon_{ij}) \ \mathrm{d}A \ , \tag{20}$$

$$T = \lim_{\Gamma^* \to 0} \int_R \frac{1}{2} \rho \dot{u}_i \dot{u}_i \ \mathrm{d}A \ , \tag{21}$$

Since the loop $\Gamma^*$ moves with the crack tip, the region $R$ is time-dependent.

In this case the dynamic energy released rate, $G_{oD}$ can be write as

$$\phi = G_{oD}(L_o, v_o, t) v_o = P - (\dot{U} + \dot{T}) \ , \tag{22}$$

For the purposes of our calculations we consider that the non-stationary energy released rate $G_{oD}(L_o, v_o, t)$ can be described by a function of kind:

$$G_{oD}(L_o, v_o, t) = G_{oD}(L_o, v_o) f(t) = G_o g\left(\frac{v_o}{c_R}\right) f(t) \ , \tag{23}$$

where $f(t \geq \tau) \to 1$

Considering that the Mott postulate [19] is valid not only at the beginning of the crack growth, at $t = 0$, but instantaneously at any time, $t$, during the whole crack growth process, i.e., while, as a crack grows with velocity $v_o(L_o(t))$. Thus the energy flux, $\phi_0(t)$, to the crack tip that is characterized by the velocity $v_o(L_o(t))$ is given by:

$$\phi_0(t) = G_{oD}[L_o(t), v_o(L_o(t)), t] \ v_o(L_o(t)) \ , \tag{24}$$

where one made use of (9). The coefficient of $v_o(L_o(t))$ is the elastodynamic energy release rate, $G_{oD}[L_o(t), v_o(L_o(t)), t]$, given by (23) [8]. Therefore

$$\phi_0(t) = G_o(L_o(t)) \ g(\frac{v_o(L_o(t))}{c_R}) \ f(t) \ v_o(L_o(t)) \ , \tag{25}$$

Based on (13) the energy flux, given in (25), can be written as

$$\begin{aligned} &G_{oD}[L_o(t), v_o(L_o(t)), t]\ v_o(L_o(t)) \\ &= G_o(L_o(t))\ g\left(\frac{v_o(L_o(t))}{c_R}\right)\ f(t)\ v_o(L_o(t))\ . \end{aligned} \tag{26}$$

For the particular case of a semi-infinite plane plate under Mode I loading the energy flux to the crack tip is derived from (14) as

$$\phi_0(t) = \frac{2\gamma_{\text{eff}}(\mathrm{d}L/\mathrm{d}L_o)}{1 - \frac{v_o}{c_R}(\mathrm{d}L/\mathrm{d}L_o)} f(t) v_o(t)\ . \tag{27}$$

### 2.6 Advanced Considerations Based on Fractal Aspects of Fracture Surface for Dynamic Fracture Mechanics

In a fast crack growth experiment as that performed by Fineberg and Gross the oscillations produced in the crack growth velocity can make that it reaches velocities close to the values of Rayleigh waves speed, ($v_o \to c_R$), and when the correspondent energy injected into the crack tip is above this value, it is enough to create new paths for the crack generating branchings [1,2]. Starting from there, if to continue having an increase in the energy injected into the crack tip due to an indefinite loading stress, that is to say, $G(t) \to \infty$, the instability and branching process stays and it starts to happen in different scales, i.e., for each new crack made by branching, being obtained a self-affine geometric pattern (invariant by scale transformation). This way it is observed that the fractal nature of instability and of branching it is nothing else than a physical confirmation that the phenomenology of the process described above continues reproducing in scale, indefinitely, while there is an energy excess into the crack tip. It can also be understood, that the chaotic nature of the fracture possesses a kind of "phenomenological memory" that repeats in different scales being registered in the fractality of fracture surfaces generated in the dynamic crack growth.

Based in the Fineberg experimental results it can be assumed that the temporal dependence of rugged crack growth of crack length has a structured self-similar [8] or self-affine fractal behaviour [17, 20, 26, 27] in the time, such as, an appropriated function satisfying the relaxation process can be supposed as,

$$L[L_o(t+\tau)] = h[L(L_o(t))]\ , \tag{28}$$

where $\tau$ is a time lag with magnitude of the order of the viscoelastic relaxation time in the sample. This hypothesis means that the crack length generated in the dynamic fracture process elapses on itself.

For the crack growth velocity to be in agreement with (28) another appropriated self-affine equation can be written as

$$v_o[L_o(t+\tau)] = v_o[L_o(t)] \frac{\mathrm{d}h[L(t)]}{\mathrm{d}L_o(t)} \frac{\mathrm{d}L_o(t)}{\mathrm{d}L(t)} \,. \tag{29}$$

where $v_o[L_o(t)] = v[L(t)]\mathrm{d}L_o(t)/\mathrm{d}L(t)$.

This mathematical procedure, naturally, introduces in the equations of dynamic fracture the two necessary conditions to describe the instability process. To know: (i) a decoupling of the dynamical functions between the input and output of system which are the elastodynamic energy release rate, $G_{oD}$, and work of fracture, $\Gamma_0$, by means of the dynamical variables of length, $L_o$, and crack growth velocity, $v_o$, [28] as it is shown in (28) and (29) and (ii) the existence of at least two situations equally probable [20], unifying them into a single condition. This second condition will be focused latter on.

One observes that, while the input energy flux into the crack tip, $\phi_0(t)$, given by (25) depends on the the crack growth velocity function, $g(\frac{v_o}{c_R})$, the output energy flux, $\psi(t)$, must depend on the local ruggedness created instantaneously in the crack growth process. Therefore, the response in the form of fracture surface, designated by $\psi(t+\tau)$, as being the energy adsorbed to form the rugged crack (shown in Fig. 2), can be written as

$$\begin{aligned} \psi(t+\tau) &= \frac{\mathrm{d}U_\gamma[L(t),v(t)]}{\mathrm{d}L(t)} \frac{\mathrm{d}L(t)}{\mathrm{d}t} \frac{\mathrm{d}h[L(t)]}{\mathrm{d}L(t)} \\ &= \frac{\mathrm{d}U_\gamma[L(t),v(t)]}{\mathrm{d}L(t)} \frac{\mathrm{d}L(t+\tau)}{\mathrm{d}t} \,, \end{aligned} \tag{30}$$

Since $\psi(t+\tau) = \psi_0(t+\tau)$, due to the energetic equivalence among the rugged and the projected crack path, then one can express (30) as

$$\psi_0(t+\tau) = \Gamma_0[L_o(t), v_o(L_o(t))] v_o[L_o(t+\tau)] \,, \tag{31}$$

where $L(t+\tau)$ is the ruggedness (i.e. actual) fracture surface length in the time, $t+\tau$, and $L_o(t+\tau)$ is its corresponding plane projected length on the direction of crack growth.

Substituting (14) into (31), one gets

$$\psi_0(t+\tau) = \frac{2\gamma_{\mathrm{eff}}(\mathrm{d}L/\mathrm{d}L_o(t))}{1 - \frac{v_o[L_o(t)]}{c_R}(\mathrm{d}L/\mathrm{d}L_o(t))} v_o[L_o(t+\tau)] \,, \tag{32}$$

This equation describe the dissipation rate in the formation of the fracture surface formation in the time $t+\tau$.

### 2.7 Dynamic Fracture Model with a Time Delay

Sharon & Fineberg [13] admit that theory predicting the motion of a crack is governed by the balance between the energy flux into the crack tip, $\phi_0(t)$, and the dissipation rate, $\psi_0(t)$, here given by (31) including the time delay between them.

$$\phi_0(t) = \alpha\psi_0(t) \;, \tag{33}$$

where $0 < \alpha < 1$ is a conversion factor of elastodynamic energy into surface energy (work of fracture).

However it is necessary to take into account the relaxation process at the crack tip. Observe that (26) says that the energy balance between the instantaneous input flux and stationary flux at crack tip, in accordance with Irwin–Mott's (10), can be written in a general form as *Stationary Flux(time, t) = Input Flux(time, t)/Coupling function of time,* $f(t)$ or

$$\phi_{o(\text{stationary})}(t) = \frac{\phi_0(t)}{f(t)} \;, \tag{34}$$

where from (23), (24) and (25) we have:

$$\phi_{o(\text{stationary})}(t) = G_{oD}[L_o(t), v_o(L_o(t))]v_o[L_o(t)] \;, \tag{35}$$

depending on the projected crack length, $L_o(t)$, already formed in each instant, $t$.

Substituting (35) into (26) one gets:

$$\begin{aligned} &G_{oD}[L_o(t), v_o(L_o(t))]v_o[L_o(t)] \\ &= G_o(L_o)g\left(\frac{v_o(L_o(t))}{c_R}\right)v_o[L_o(t)] \;. \end{aligned} \tag{36}$$

The particular experimental apparatus in use influences the functional form of the kinetic energy, $T_o(L_o, v_o)$ developed by the cracks and this in turn affects the ruggedness developed instantly in the fracture. Therefore it should exist a relationship between the response in the form of a local ruggedness created instantaneously in the crack growth process and the of the crack growth velocity function, $g(\frac{v_o}{c_R})$, Thus, assuming a function $\frac{dh(L(t))}{dL_o(t)}\frac{dL_o}{dL} \sim 1/[g(\frac{v_o(L_o(t))}{c_R})]$ the energy flux, given by (26), can be written as

$$\begin{aligned} &G_{oD}[L_o(t), v_o(L_o(t))]\ v_o[L_o(t)]\frac{\mathrm{d}h[L(t)]}{\mathrm{d}L_o(t)}\frac{\mathrm{d}L_o}{\mathrm{d}L} \\ &= \alpha G_o[L_o(t)]v_o[L_o(t)] \;. \end{aligned} \tag{37}$$

Observe from (37) that the Mott postulate, enunciated before, becomes valid instantaneously during the whole crack grow process by means of a new insight in the energy flux balance with time delay. Therefore from the elastodynamic crack growth condition given in (11), and using (29) in (37), this equation can be expressed as

$$\Gamma_0[L_o(t), v_o(L_o(t))]v_o[L_o(t+\tau)] = \alpha G_o[L_o(t)]v_o[L_o(t)] \;. \tag{38}$$

In accordance with the self-affine hypothesis made in (28) and (29) the energy flux balance between the input and output with a time delay, $\tau$, may be written as $\alpha$. *Relaxed Flux(time past, t) = Output Flux(time present = time past plus relaxation time, t + τ)*, or in accordance with Irwin–Mott's (10), (38) can be written in a general form as

$$\phi_{o(\text{relaxed})}(t) = \frac{1}{\alpha}\psi_0(t+\tau) \; . \tag{39}$$

Therefore at time, $t$, the flux of elastodynamic energy into the specimen to the crack tip, $\phi_0(t) = G_{oD}[L_o(t), v_o(L_o(t)), t] \; v_o[L_o(t)]$, will create in front of the crack tip, at time $t+\tau$, the conditions for the formation of a process zone (viscoelastic region of size $\sim l_o$) that will separate at time, $t+\tau$, forming a rugged crack having length $L(t+\tau)$, necessary for crack growth (e.g. breaking of chemical bonds, formation and nucleation of dislocations, etc.). This theoretical result resumes all the problematics found by Fineberg and Gross [1, 2], already explained in the Sects. 2.1 and 2.2 of this chapter.

## 2.8 Chaotic Nature of Dynamic Fracture

The chaotic nature of fractures can be revealed by different forms depending of the particular experimental set up. For the case of a semi-infinite plane plate under Mode - I loading the function $g(v_o[L_o(t)]/c_R)$ determines a logistic map well known in the literature concerning chaos theory [29, 30]. Therefore, using (14) in (38) or (41) and (32) in (39) this becomes,

$$\begin{aligned} &\alpha G_o[L_o(t)]v_o[L_o(t)] \\ &= \frac{2\gamma_{\text{eff}}(\mathrm{d}L/\mathrm{d}L_o(t))}{1-\frac{v_o[L_o(t)]}{c_R}(\mathrm{d}L/\mathrm{d}L_o(t))} v_o[L_o(t+\tau)] \; . \end{aligned} \tag{40}$$

Rewriting (40) and multiplying the resulting by $\mathrm{d}L/\mathrm{d}L_o$ term it yields the following expression for the normalized crack growth velocity $v_o[L_o(t+\tau)]$ corresponding to projected surface

$$\begin{aligned} &\frac{v_o(L_o(t+\tau))}{c_R}\frac{dL}{dL_o(t)}\bigg|_{t+\tau} \\ &= \frac{\alpha G_o(L_o(t))}{2\gamma_{\text{eff}}(\frac{dL}{dL_o(t)})} \\ &\quad \times \frac{v_o(L_o(t))}{c_R}\frac{dL}{dL_o(t)}\left[1-\frac{v_o(L_o(t))}{c_R}\frac{dL}{dL_o(t)}\right] , \end{aligned} \tag{41}$$

which has the form of the equation of the logistic map (Fig. 3) [29–31]. For convenience, instead of the delay in time, $\tau$, (41) can be written, as a sequence of events, i.e.,

$$x_{ok+1} = \mu_0 x_{ok}(1 - x_{ok}) \; , \tag{42}$$

where $x_{ok}$ corresponds to the normalized projected surface crack growth velocity and the coefficient, $\mu_0$, to control parameter of (31), i.e.,

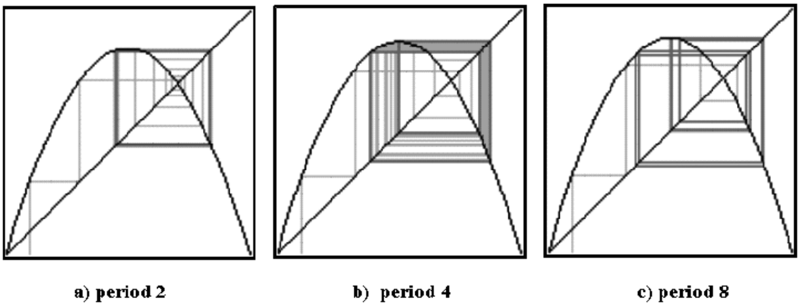


**Fig. 3.** Logistic map of the logistic equation $x_{ok+1} = \mu_0 x_{ok}(1 - x_{ok})$ related to the input energy flux, $\phi_0(t)$, and output flux of dissipated power, $P_0(t+\tau)$ in function of the normalized velocities $v_o(t)/c_R$ and $v_o(t+\tau)/c_R$, showing the cycles or periods of iteration

$$\mu_0 \equiv \frac{\alpha G_o(L_o)}{2\gamma_{\text{eff}}(\mathrm{d}L/\mathrm{d}L_o)} , \tag{43}$$

$$x_{ok} \equiv \frac{v_o[L_o(t)]}{c_R} \frac{\mathrm{d}L}{\mathrm{d}L_o(t)} , \tag{44}$$

and

$$x_{ok+1} \equiv \frac{v_o[L_o(t+\tau)]}{c_R} \frac{\mathrm{d}L}{\mathrm{d}L_o(t)} , \tag{45}$$

respectively.

In Fig. 4, the iterations of this equation are shown as a function of the control parameter, $\mu_0$, [30] which can be identified with the crack growth characteristics.

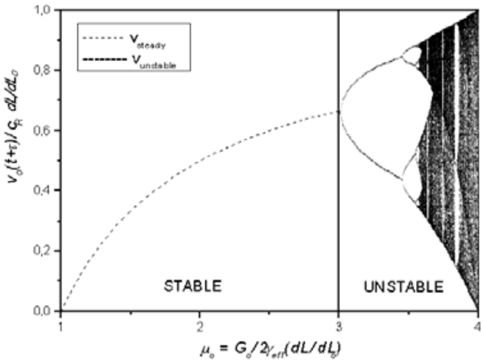


**Fig. 4.** Iterates of the logistic map of (48) as a function of $\mu_0$ for $1.0 \leq \mu_0 \leq 4.0$. An transient of 200 points has been discarded in each case [30]. The control parameter, $\mu_0$, is given by (43) and $v_o(t+\tau)/c_R$ is the normalized velocity as it was explained in the text

Observe that a normalized crack growth velocity to rugged surface, defined as $v(L(t)) = v_o[L_o(t)]\mathrm{d}L/\mathrm{d}L_o(t)$, can also be written in the logistic equation form, in analogous way to plane projected fracture surfaces, dropping all the zero subscripts in the (42), (43) and (44) in a such way that the new rugged crack growth velocity can be written as

$$\frac{v[L(t+\tau)]}{c_R} = \frac{\alpha G(L)}{2\gamma_{\mathrm{eff}}} \frac{v[L(t)]}{c_R} \left(1 - \frac{v[L(t)]}{c_R}\right) , \qquad (46)$$

where $G(L)$ is the elastic energy release rate to the rugged crack path, giving place to write $x_{k+1} = \mu x_k(1 - x_k)$. In analogous way it gets $x_k = v[L(t)]/c_R$ and $\mu = \alpha G(L)/2\gamma_{(\mathrm{eff})}$. This means that the same conditions can be repeated at each new crack path created, in a indefinite branching process, originating a self-affine spatial fractal pattern formed by all cracks generated during the branched crack growth process.

### 2.9 The Instability Process Under the Sight of Time Delay

Therefore, the instability dynamic process on the fast crack growth and crack branching in brittle materials like the soda-lime glass and PMMA can be explained in the following way: The existence of a time delay, $\tau$, between the energy flux injected into the crack tip, $\phi_{o(\mathrm{inst})}$, (25), and the spent energy flux (or the dissipated power, $\psi_0$, (31)) to form the fracture surfaces, (32), (Fig. 3), produces from a critical velocity, (44), a "uncompassing" (Fig. 3) between the crack growth velocity, $v_o(t)$, and the rate of formation of the fracture surfaces, $v_o(t+\tau) = \mathrm{d}A_o(t+\tau)/\mathrm{d}t$. This "uncompassing" is responsible for an indetermination in the crack growth velocity, (Fig. 4), at crack tip that by its turn gives rise to a dynamic instability in the form of oscillations in the crack growth velocity. This instability produces a rugged fracture surface increasing the consumption of energy. This increase limits the crack growth velocity to a smaller value than the speed of Rayleigh waves in the material, ($v_o \leq c_R$), (46), producing a new delay in relation to fracture surfaces formation rate and increasing still more the "uncompassing" between the injected energy flux and the worn-out energy rate to form the fracture surfaces and so forth (Figs. 3 and 4).

## 3 Results

Equations (45) and (41) can be explored through the graphs of $\psi_0(t+\tau)$ and $\phi_0(t)$ versus $v_o(t)$ (Figs. 3 and 4) and of $v_o(t+\tau)/c_R$ versus $\mu_0$ (Fig. 4). By means of this diagram one notices that combining the magnitudes, $\phi_0(t)$ and $\psi_0(t+\tau)$ it generates an instability whenever a temporal delay exists among them. Actually, (42) is the well-known logistic equation. However, Before analyzing Fineberg et al. experimental results [1] we start from the use of

Slepyan's criterion, given by Maximum Energy Dissipation Principle [20], and modify it in order to include the ruggedness term, $\mathrm{d}L/\mathrm{d}L_o$, of fracture surface [24]. From this result it was possible to obtain the energy balance between the input and output of the system, where in our notation it is given by (46). Observe that this expression can be written in the following way:

$$\alpha G_o[L_o(t)]\left(1-\frac{v_o[L_o(t)]}{c_R}\frac{\mathrm{d}L}{\mathrm{d}L_o(t)}\right)v_o[L_o(t)] = 2\gamma_{\mathrm{eff}}\frac{\mathrm{d}L}{\mathrm{d}L_o(t)}v_o[L_o(t+\tau)]\,. \tag{47}$$

Although its left side reminds the expression of the classical result proposed by Freund [8] where the elastodynamic energy release rate being, $G_{oD} = G_o[1 - v_o(t)/c_R]$, for a medium with infinite extension ((8.77) of Freund [8]). Therefore, a first result that must be observed in this paper is that the classical dynamic fracture mathematical formalism is reobtained from (41) when the relaxation process is almost negligible and the persistent creep phenomenon is not considered into a thin crack tip in short range effect described by punctual process zone. In this case it has that $v_o(t+\tau) = v_o(t)$ and therefore the classical comes out:

$$\frac{v_o(t)}{c_R} = \left(1-\frac{2\gamma_{\mathrm{eff}}}{G_o(L_o)}\frac{\mathrm{d}L}{\mathrm{d}L_o(t)}\right)\frac{\mathrm{d}L_o(t)}{\mathrm{d}L}\,. \tag{48}$$

Whenever the elastic energy release rate, $G_o(L_o)$, increases linearly with the crack length, $L_o$, which in the absence of ruggedness, i.e. $\mathrm{d}L/\mathrm{d}L_o = 1$, (48) becomes,

$$v_o = c_R\left(1-\frac{L_{oc}}{L_o}\right)\,. \tag{49}$$

This result shows the stable region of the fracture process formed by "fixed point of order one" of the logistic map (Fig. 4) where the conditions of the crack growth are considered to be slow because the viscoelastic properties of the material do not influence the process.

On the other side, the first graph (Fig. 3) shows the dependence of elastodynamic energy flux, $\phi_0(t)$, given by (43) and (44), that flows into the crack tip, and of the dissipated power spent to form the fracture surface, $\psi_0(t+\tau)$, given by (31) and (48), in function of the normalized crack growth velocities $v_o(t)/c_R$ and $v_o(t+\tau)/c_R$ respectively. This graph shows that the energy flux, $\phi_0(t)$, possesses a maximum value at $v_o/c_R = 0.5$ and a nonlinear dependence of dissipated power, $\psi_0$, on the normalized velocity $v_o(t+\tau)/c_R$ as it was proposed by the model (26).

The second graph (Fig. 4) shows the iterations of the logistic map, (41) and (48), of the normalized crack velocity, $v_o(t+\tau)/c_R$, (44) as a function of the control parameter, $\mu_0$, given by (43). This control parameter, $\mu_0 = \frac{G_o}{2\gamma_{\mathrm{eff}}(dL/dL_o)}$, for a medium with infinite extension, is linearly proportional to the crack length, $L_o(t)$; then as the crack grows, the control parameter increases. Therefore this graph represents the dependence of the speed with the retardation, $v_o(t+\tau)$, in function of the projected crack length, $L_o$. For values of $1 \leq \mu_0 \leq 3$ the crack growth process is dynamically stable since at those speeds the viscoelastic process zone in front of the crack tip is too small to influence the process. However, the time delay that exists between the energy that flows into the crack tip, $\phi_0(t)$, and the dissipated power for crack growth, $\psi_0(t+\tau)$, causes instabilities that show up in the interval $3 \leq \mu_0 \leq 4$. Such instability begins only when the control parameter reaches the value, $\mu_0 = 3$. Exactly at this point, $\mu_0 = 3$, there is a flip bifurcation [29–31] and it begins the influence of the time delay (where $\tau \sim l_o/v_o$) into the process zone causing instabilities in the crack growth. As shown in Fig. 4 at $\mu_0 > 3$ the fixed points determined by (48) are not stable anymore. Considering that the relaxation with time delay is fixed it is seen that the length of the process zone oscillates in time becoming responsible for the oscillations in the crack growth velocity, $v_o(t)$. The velocity function, $v_o(t+\tau)$ in (41), which is related to the dissipated power, $\psi_0(t+\tau)$, to $\mu = 3$, has the value equal to $2/3c_R$. This value from (48) corresponds to a critical velocity

$$v_{o(\text{critic})} = 1/3c_R \ . \tag{50}$$

The main assumption of a time delay is expressed mathematically by formula (48) where the factor $\mu_0$ of (43) is taken to be fixed. This means that the crack growth velocity at $t+\tau$ is proportional to the energy flux into the growing crack tip at time $t$. This relationship is very interesting since it leads to the oscillating type of instability for the steady motion (with $v_{o(\text{steady})} = c_R(1-1/\mu_0)\mathrm{d}L_o/\mathrm{d}L$ as the solution (48) if $3 \leq \mu_0 \leq 4$. By the way, if $\mu_0$ increases (from $\mu_{oc} = 3$ to a limit value $\mu_{o\,\max} = 4$), the minimum limit of the averaged speed, $v_{o\,\min}$, is given approximately by

$$\frac{v_{o\,\min}}{c_R} = \left(1 - \frac{1}{3}\right)\left(\frac{\mathrm{d}L_o}{\mathrm{d}L}\right)_{\max} = \frac{2}{3}\left(\frac{\mathrm{d}L_o}{\mathrm{d}L}\right)_{\min} , \tag{51}$$

just depending on the maximum value of ruggedness, $\mathrm{d}L_{\min}/\mathrm{d}L_o$, in this point. For a value of ruggedness equal to, $\mathrm{d}L/\mathrm{d}L_o = 3/2$ the limit of the averaged speed, $v_o$, is given by:

$$v_o = 0.44\, c_R , \tag{52}$$

and the maximum limit of the averaged speed, $v_{o\,\max}$, is given approximately by

$$\frac{v_{o\,\max}}{c_R} = \left(1 - \frac{1}{4}\right)\left(\frac{\mathrm{d}L_o}{\mathrm{d}L}\right)_{\max} = \frac{3}{4}\left(\frac{\mathrm{d}L_o}{\mathrm{d}L}\right)_{max} , \tag{53}$$

depending also on the maximum value of the ruggedness, $\mathrm{d}L_{\max}/\mathrm{d}L_o$, at this point. Equally for a value to ruggedness value equal to $\mathrm{d}L/\mathrm{d}L_o = 3/2$ the limit of the averaged speed, $v_o$, is given by:

$$v_o = 0.5\, c_R \ , \tag{54}$$

from where the normalized crack velocity $v_{ok}/c_R$ saturates at $c_R(\mathrm{d}L_o/\mathrm{d}L)_{\max}$, below of Rayleigh waves velocity, which is approximately the saturation velocity measured by Fineberg et al. [1,27].

As can be seen in Fig. 3, $v_{0\,\max}$ and consequently $G_{oD}$ has tendency to approach the maximum of the energy flux. After $\mu_0 \geq 4$ there is no sense to speak about the influence of $\mu_0$ parameter over the crack growth because the $G_{oD}$ reaches the maximum constant flux and $v_o$ reaches a maximum constant velocity and after this the crack growth follows a new stationary state conserving the ruggedness already created to maintain the this maximum values. It can also be observed that (53) assumes an infinite body sample where the influence of external stress field it is negligible and after the maximum flux condition have been reached the dynamic energy release rate, $G_{oD}$, does not depend anymore on the sample length.

### 3.1 Comparison Between Theory and Experiment

Fineberg et al. [1,2] observed that at the onset of instability the fracture surface changes from a featureless (on a scale larger than 1 µm) to a jagged structure which develops into coherent oscillations and coalesce downstream of the crack growth broadening to extend over the entire width of the sample. This change in morphology as the crack accelerates can be associated with the geometric increase in the number of flip bifurcations, or, with the number of non-trivial stable fixed points as can be seen in Fig. 4. To each stable fixed point corresponds a given velocity $v_{ok+1}/c_R$. As the crack grows the straight line crack velocity oscillates among the allowed velocities, looking for alternative paths of energy dissipation in excess at the crack tip by means the generation of a rugged surface that mathematically correspond to more energetic path. It is therefore reasonable to associate the issuing increase of ruggedness with the number of possible velocities. At $\mu_0 < 3$ there is only one possible velocity (one stable fixed point on the logistic map) and therefore the surface is smooth. As the crack propagates $\mu_0$ (given by (43)) increases with the crack length, $L_o(t)$, and the number of allowed straight line crack velocities becomes larger and larger. The straight line crack velocity oscillates among the allowed values (projections of the actual velocity) resulting in morphologies of higher complexity, in agreement with the observations of Fineberg et al. [1,2].

Peter Gumbsch [32–34], using molecular dynamics methods in your simulations, reported that the instability in dynamic crack growth sets at $G_o/2\gamma_{o(\mathrm{eff})} = 3$. This condition for initiate initiating instabilities, considering $\alpha \mathrm{d}L/\mathrm{d}L_o \simeq 1.0$, is equivalent to $\mu_0 = 3.0$, as is given by (43). Therefore this result is in reasonable agreement with the result shown by the chaotic model presented in this paper.

Taking for the energy conversion factor $\alpha = 1.0$ one obtains from (48) the value of 0.33 for the normalized crack velocity at which instabilities should occur. However, Fineberg and co-workers observed that beyond a critical value of 0.34 the normalized velocities, $v_o/c_R$, start to show instabilities [1,2]. Therefore this theoretical value of $0.33\,c_R$ is in excellent agreement with the value of experimental result of 0.34 [1, 2], measured by Fineberg–Gross et al., at the moment the instability starts. Gross [22] also observed that a visible rough structure (branching) appears near from $0.42\,c_R$ in the form of a parabolic crack branching.

## 4 Discussion

The experiments performed by Fineberg and co-workers [1, 2] provide evidence for instability in the brittle fracture of isotropic materials. On the other hand, theories based on conventional concepts such as energy balance [16] and quasi-static configurational forces at crack tips show no indication of strong oscillatory or branching instabilities [35]. Yoffe [36] by analyzing the stresses in the neighborhood of a crack tip growing at high velocity hinted about the emergence of instabilities but the analysis is not a truly dynamic theory of forces and accelerations of fractured surfaces.

The basic property of dynamic fracture mechanics is that the processes near the fault tip occur at near wave velocities, and for this reason the crack tip is independent of the details of loading. This is a famous theorem proved independently by Kostrov and Eshelby in 1964 and 1969, respectively, for the antiplane case and by Kostrov and Nikitin in 1970 for general loading. Observe that (17) refers to a particular experimental set up. Therefore, in accord to the functional dependence of this equation for the $g(v_o/c_R)$ term or depending on of the particular form of the experiments other kinds of logistic maps can be obtained, since that the same procedure of calculations accomplished until now can be done. Equation (46) developed in this paper is equivalent to (12), and the same improvements (finite size of the sample, influence of boundary, etc.) proposed to (12) [8] can be incorporated into (43) and (46) without consequences on the results presented in this paper. This is corroborated by experimental evidence [1, 2, 13] showing that the onset of instabilities is independent of the size of the sample and/or of the geometrical set up of the experiment (see also [3]).

## 5 Summary and Conclusions

This chapter presents arguments in favour of chaotic behaviour of rupture. The arguments are general and based on energy conservation principle which are totally valid in fracture mechanics. The central hypothesis is the energy flux through the crack tip is converted there into fracture energy with a time delay, $\tau$, due to the development of a viscoelastic process zone in front of the crack tip. It is tacitly assumed in this paper that such a delay exists and it has a well defined time scale, $\tau$, being a characteristic property of the material. Its magnitude is of the order of the viscoelastic relaxation time of the sample material under local fracture conditions. A key assumption of the theory is that the onset of instability observed in the velocity of dynamic crack growth is due to the time delay, which yields (41) and (46). This time delay factor, $\tau$, in (28) implies the possibility to derive an equation for the crack growth the velocity in the form of a logistic map equation.

Before concluding this chapter it is necessary to notice that the hypothesis of linear energy transfer as given by (46) and (45) is an oversimplified approach. The energy release rate, $G_o$, is linearly dependent on the crack length whereas the crack resistance, $R_o$, rises in a non-linear form [10, 12]. Based on (28), (29) and (41), this property will be used in a forthcoming paper in which it is shown that the energy dissipation can also be written in the form of a logistic map having as consequence crack branching and other phenomena so far not explained by the classical fracture theory.

The purpose of this chapter is to show that contrary to what has been thought previously, the most familiar models in fracture mechanics are intrinsically incomplete. Therefore, it was used an as simple as possible case of dynamic crack growth of a semi-infinite body with plane strain condition, and well established concepts and results, to derive an expression for the crack velocity in the well known form of a logistic equation and map. From this map conclusions regarding instabilities of crack growth are drawn and compared with the experimental results obtained by Fineberg and co-workers [1]. This work shows that other logistic maps can be built, accordingly to the particularity of the experiment and accordingly to the expression of its the kinetic energy. This article presents a new picture for dealing with fracture dynamics, making use of logistic maps as a new method for predicting the possible velocities that a crack can reach and from there to try to reproduce its geometric fractal behaviour, ruggedness, etc. Each particular material and each particular experimental testing condition will determine the type of map and the type of crack as well. The logistic map built has an interpretation which enables to understand even more complex situations for the phenomenon under study.

From the above results it is concluded that the instabilities involved in dynamic fracture are consequences of the mathematical structure of chaos that underlies such phenomena. It was possible to write the straight line crack velocity in the form of a logistic map explaining the onset of instabilities observed by Fineberg et al. [1]. This achievement brings into fracture mechanics all the mathematical structure developed for complex systems. This theoretical approach provides a single and concise tool to determine among others properties the conditions under which crack growth becomes dynamically unstable and branching takes place as will be shown in a forthcoming paper.

The literature usually shade that there are fractals in the quasi-static fracture of surfaces. Then undoubtedly, in fracture dynamics there will be chaotic behaviour in the formation of the same ones. Therefore, if the model proposed in this chapter is not the final answer for the subject, at least it is an initial step, it lifts and it opens a new proposal for studying fracture dynamics. Therefore, we want to say that experimental research is needed to illuminate the theoretical evidences more closely.

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